

Chromaticity in Gaia-3

Jos de Bruijne*, Lennart Lindegren, Oskar Svensson, Carme Jordi, Francesca Figueras, Anthony Brown

*Astrophysics & Fundamental Physics Missions Division (SCI-SA)

SCI-SA / RSSD / ESTEC / ESA

Postbus 299

NL-2200 AG Noordwijk (ZH)

The Netherlands

Jos.de.Bruijne@rssd.esa.int

GAIA-CA-TN-ESA-JDB-028-1

August 4, 2006

Abstract This document presents an exploratory study of chromaticity calibration models for Gaia-3, currently under development at Lund Observatory. The novel approach presented here is based on knowledge of the object spectrum coming from BP/RP rather than fluxes from BBP filter bands. With a simple calibration model, the calibration residuals are typically 3% of the photon-statistical centroiding error σ_ξ for a single CCD for stars fainter than 15-th magnitude and 6–10% at the bright end ($G \lesssim 13$ mag). These results provide confidence that, with the Gaia-3 design, an operational chromaticity calibration procedure can be developed for usage in the Gaia data processing such that MRD requirement SCI-340 is met.

1 Introduction

1.1 Chromaticity

One of the most fundamental biases affecting Gaia’s astrometric centroid measurements is related to *chromaticity*. Although the astrometric instrument contains no refractive optics, the images are slightly chromatic because of the wavelength dependence of diffraction. Any wavefront aberration that is an odd function of the along-scan pupil coordinate (apart from a simple wavefront tilt) will produce an asymmetric diffraction image, and the width, shape, and position of it will vary with wavelength. As a result, the centroid of the Line Spread Function (LSF) will depend on the actual spectral energy distribution (SED) of the object, folded with the instrument response function. This effect is known as chromaticity.

With the kind of wavefront aberrations expected for Gaia, the relative displacement between an early-type star and a very red star may be of order 1 mas, or 20 times higher than the photon-statistical noise centroiding noise for a bright star. It is thus necessary to eliminate chromaticity to a high degree by careful calibration. This calibration is performed — for each individual object — in the on-ground data processing.

1.2 Chromaticity calibration

Up to the selection, in early 2006, of EADS-Astrium as the Gaia prime contractor, it was always planned to correct for the chromaticity bias, which systematically affects all objects crossing Gaia’s astrometric focal plane, using a set of broad-band photometric measurements (loosely referred to as the BBP fluxes). Several studies on this topic were made, most by Lennart Lindegren (see, for example, his notes GAIA-LL-016, GAIA-LL-024, GAIA-LL-039, GAIA-LL-049, GAIA-LL-053, and GAIA-LL-064; see also GAIA-ML-015, GAIA-ARI-BAS-002, GAIA-CUO-137, and 2006A&A...449..827B). In these notes — and in GAIA-LL-039 in particular — Lindegren demonstrated that simple, linear or quadratic, regression models using BBP

fluxes can be used to predict (and thus correct) chromatic centroid shifts with a precision of about 1%.

The ESA performance-calculation guidelines, enforced upon the Gaia prime contractor as official procedure to verify compliance with the Mission Requirements Document (MRD) science-performance requirements, say the following on the issue of chromatic corrections: "It shall be demonstrated [by the prime contractor] (in consultation with the Gaia Science Team) that appropriate (broad-band photometric) measurements will be available for each observed object to allow a chromaticity correction with a random error $\leq 1\%$ ". This 1% random error was taken from Lindegren's Gaia-2 (BBP) study GAIA-LL-039. The MRD itself, moreover, requires that the calibration of chromatic effects shall be feasible "with end-of-mission residuals of less than $2.5 \mu\text{as}$ for objects with $V = 15 \text{ mag}$ and $30 \mu\text{as}$ for objects with $V = 20 \text{ mag}$ " (see SCI-340).

With the selection of EADS-Astrium as Gaia prime contractor, the Gaia flight payload design no longer features broad-band photometric measurements but low-dispersion spectra, obtained in the Blue and Red Photometers (BP and RP). EADS-Astrium, so far, have simply *assumed* that these spectral data will also allow to calibrate chromatic residual shifts in AF with a precision of 1%. Formally, however, the 1% precision has been demonstrated to be applicable to BBP-type designs only (Gaia-2), and not necessarily for the case of low-dispersion photometric measurements (Gaia-3).

1.3 Aim of this document

The aim of this note is to collect the currently available information and present a first, exploratory investigation of the potential precision with which chromatic centroid shifts can be calibrated in Gaia-3. This note merely presents a sensitivity analysis and is not 'the final word' on this topic and neither presents the definitive method with which chromatic centroid shifts will be calibrated in the Gaia data processing pipeline. Based on the initial and indicative results presented here, more detailed and realistic studies will have to be initiated within CU3 and CU5, with the ultimate aim of developing a robust operational procedure that can be used in the Gaia data processing. Formally, this note responds to the action present under Review Item Discrepancy (RID) PL-0032 of the Gaia System Requirements Review (SRR), conducted in June–July 2006, which asks ESA to confirm that the 1% precision currently assumed by Astrium is a realistic chromaticity-calibration residual for Gaia-3.

2 A novel approach for chromaticity calibration

Recently, Lennart Lindegren and Oskar Svensson (Lund Observatory) have been working on chromaticity calibration starting from a knowledge of the spectrum rather than the (BBP) filter bands (see page 10 of Lindegren's GST17 presentation, available on the GaiaWiki). Details of their findings will be reported elsewhere.

In short, Lindegren's and Svensson's findings suggest that a much simpler calibration procedure than was anticipated before will be feasible. It appears namely possible to calculate, from the BP/RP data, an 'effective wavelength' λ_{eff} such that the chromatic displacement can be modeled (and thus corrected for) as a function of λ_{eff} only. The optimum definition of λ_{eff} depends on the centroiding procedure, and in particular on the relative weight of the core and wings of the LSF. However, it appears that the recipe for λ_{eff} can simply be a photon-weighted generalised mean (also known as Hölder mean or power mean), with exponent q in the range 0 to -3 (see, for example, http://en.wikipedia.org/wiki/Generalized_mean).

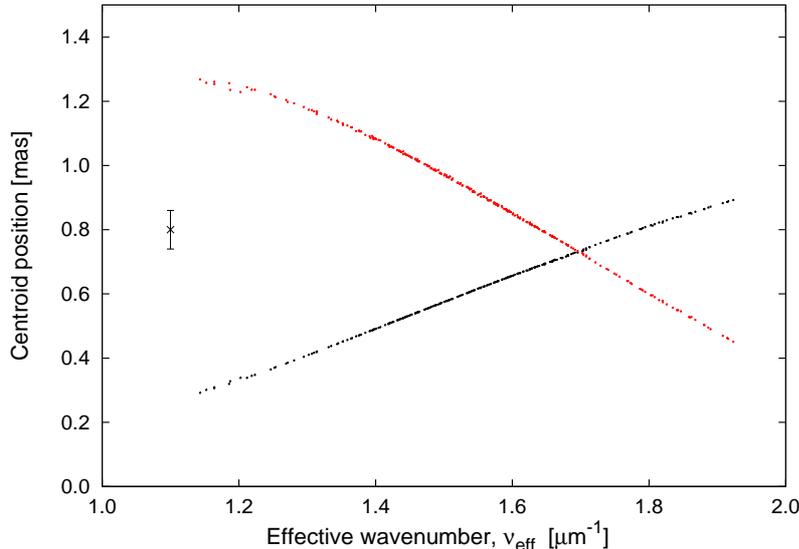


Figure 1: The dots show the computed centroid positions for the polychromatic diffraction images of different spectra versus effective wavenumber ν_{eff} , for two representative (Gaia-3) wavefront-error maps. Stellar spectra are taken from the Pickles library, without and with interstellar extinction ($A_{550} = 2$ mag). The error bar shows the photon-statistical centroiding error σ_{ξ} for a single-CCD transit of a bright star ($G \lesssim 13$ mag).

3 Chromaticity calibration in Gaia-3

Figure 1 shows the variation of the polychromatic centroid position¹ with the inverse of the effective wavelength, calculated with $q = -1$, for all stars in the Pickles stellar library (with extinctions $A_{550} = 0$ and 2 mag) and two arbitrary Gaia-3 WFE maps (F32 for CCD strip AF5, CCD row 4 in black and F55 for CCD strip AF1, CCD row 7 in red).

Figure 1 shows that the chromatic displacement is almost a unique (and roughly linear) function of $\nu_{\text{eff}} = \lambda_{\text{eff}}^{-1}$, where ν_{eff} is the ‘effective wavenumber’:

$$\nu_{\text{eff}} = \frac{\int \phi_{\lambda} \lambda^{-1} d\lambda}{\int \phi_{\lambda} d\lambda}, \quad (1)$$

where ϕ_{λ} is the detected photon-flux distribution per unit wavelength.

The effective wavenumber can be estimated from the BP/RP photometric observations and the chromaticity can then be modeled by means of a low-order polynomial in ν_{eff} . The coefficients of this polynomial are, in general, functions of the field index (f) and the position within the field (CCD index n and across-scan pixel coordinate μ), and may also have some variation with time. A reference wavenumber ν_0 must be adopted to fix the zero point, so that the calibration model may include terms like:

$$\eta_{fk}(\mu) = \dots + C_1(\nu_{\text{eff}} - \nu_0) + C_2(\nu_{\text{eff}} - \nu_0)^2 + \dots, \quad (2)$$

where separate coefficients C_i apply for the different combinations of f , n , and intervals in μ (η denotes the along-scan field angle).

¹The centroid has been defined according to the principle described in GAIA-LL-068, using Tukey’s biweight with $s = 2.7$ pixel as weight function.

However, in view of the current goal, which is to ascertain whether the MRD requirement SCI-340 can reasonably be met with *any* (as yet unspecified) calibration model, we simply fix $q = -1$, determine the statistical error by which λ_{eff} can be estimated from the BP/RP spectra, and finally translate this to a corresponding uncertainty in the chromaticity correction using the slopes of the relations in Figure 1 (assuming that the chromaticity calibration is perfect).

4 First results

Anthony Brown (Sterrewacht Leiden) provided BP and RP spectra based on the BaSeL2.2 and NextGen libraries for the current instrument design (in particular, ‘125% dispersion’ for BP, 4500 TDI lines for RP, and ‘Gaia-2’ WFE maps for BP and RP). The simulation procedure is outlined in GAIA-CA-TN-LEI-AB-005-7. Carme Jordi and Francesca Figueras (University of Barcelona) calculated ‘effective wavelengths’ for these spectra using $q = -1$, i.e., as the inverse of the photon-flux-weighted wavenumber. Details of this procedure are reported in GAIA-CA-TN-UB-CJ-040-1.

We use Eq. (4) from GAIA-CA-TN-UB-CJ-040-1 to define λ_{eff} from the BP/RP-sample data:

$$\lambda_{\text{eff}} = \left(\frac{\sum_{i \in \text{BP}} R_{\lambda_i}^{\text{BP}} B_{\lambda_i}^{\text{BP}} \lambda_i^q + \sum_{j \in \text{RP}} R_{\lambda_j}^{\text{RP}} B_{\lambda_j}^{\text{RP}} \lambda_j^q}{\sum_{i \in \text{BP}} R_{\lambda_i}^{\text{BP}} B_{\lambda_i}^{\text{BP}} + \sum_{j \in \text{RP}} R_{\lambda_j}^{\text{RP}} B_{\lambda_j}^{\text{RP}}} \right)^{\frac{1}{q}}, \quad (3)$$

where $q = -1$, $R_{\lambda_i}^{\text{XP}}$ denotes the measured flux in sample i of the XP (BP/RP) photometer, and B_{λ}^{XP} transforms the overall response in XP to the overall response in AF (see GAIA-CA-TN-UB-CJ-040-1 for details). Table 1 (based on Table 4 in GAIA-CA-TN-UB-CJ-040-1) shows the results for a series of representative (unreddened) stars, for four different magnitudes ($G = 13, 15, 18, \text{ and } 20$ mag). The Table also contains $\sigma_{\lambda_{\text{eff}}}$, which is the precision with which λ_{eff} can be determined from single-transit BP/RP measurements (based again on Table 4 in GAIA-CA-TN-UB-CJ-040-1). This precision is set by the noise on the BP/RP-sample data (in turn caused by Poisson noise of the object, Poisson noise of the sky background, and total detection noise), errors in the BP/RP wavelength calibration, and errors in the calibration of the instrument-response differences between BP/RP and AF.

With knowledge of λ_{eff} and $\sigma_{\lambda_{\text{eff}}}$, the single-CCD chromaticity residual calibration error, $\sigma_{\text{cal,chrom}}$, is readily determined as:

$$\sigma_{\text{cal,chrom}} = \frac{a \cdot \sigma_{\lambda_{\text{eff}}}}{\lambda_{\text{eff}}^2}, \quad (4)$$

where $a = 1000 \mu\text{as per } 0.0008 \text{ nm}^{-1}$ ($1000 \mu\text{as per } 0.8 \mu\text{m}^{-1}$) is the (maximum) slope of the relations in Figure 1. Table 1 also lists $\sigma_{\text{cal,chrom}}$, in units of μas .

5 Discussion

Numerous issues need further thought, attention, and study:

- Different WFE maps result in different ‘optimum’ q -values, which may range from around 0 to -3 . It remains to be investigated whether acceptable compromise values of q can be found in a min-max sense, minimising the maximum RMS value for a given range of WFE maps;
- The representativity of the two Gaia-3 WFE maps used here remains to be established;
- It remains to be demonstrated that a single value of q can characterise all kinds of spectra, including quasar spectra which are of relevance for establishing the Gaia reference frame;

Table 1: Chromaticity residual calibration error $\sigma_{\text{cal,chrom}}$ (applicable to a single AF CCD) for a series of different stars and four different G magnitudes. The columns λ_{eff} and $\sigma_{\lambda_{\text{eff}}}$ (taken from Table 4 in GAIA-CA-TN-UB-CJ-040-1) denote the ‘effective wavelength’ and the precision with which this wavelength can be determined from single-transit BP/RP data. Typical, order-of-magnitude single-CCD-transit centroiding errors σ_{ξ} are also given, for reference. At the bright end, σ_{ξ} decreases further from 80 μas at $G = 13$ mag to 60 μas at saturation.

Star	G mag	λ_{eff} nm	$\sigma_{\lambda_{\text{eff}}}$ nm	$\sigma_{\text{cal,chrom}}$ μas	σ_{ξ} μas
B5V	13	536.3	1.3	5.6	80
F2V	13	580.5	1.5	5.6	80
G2V	13	605.9	1.6	5.4	80
K3III	13	641.4	1.7	5.2	80
M0V	13	709.3	1.8	4.5	80
M0III	13	749.8	1.9	4.2	80
B5V	15	536.3	1.4	6.1	200
F2V	15	580.5	1.6	5.9	200
G2V	15	605.9	1.7	5.8	200
K3III	15	641.4	1.7	5.2	200
M0V	15	709.3	1.9	4.7	200
M0III	15	749.8	2.0	4.4	200
B5V	18	536.3	3.2	14	850
F2V	18	580.5	3.7	14	850
G2V	18	605.9	4.0	14	850
K3III	18	641.4	4.5	14	850
M0V	18	709.3	5.8	14	850
M0III	18	749.8	6.7	15	850
B5V	20	536.3	15	64	2500
F2V	20	580.5	17	64	2500
G2V	20	605.9	19	66	2500
K3III	20	641.4	23	69	2500
M0V	20	709.3	32	78	2500
M0III	20	749.8	38	84	2500

- The operational calibration model will require a regression of the centroid position against λ_{eff} (or ν_{eff}) using a low-order polynomial (Section 3);
- The assumptions made in GAIA-CA-TN-UB-CJ-040-1, notably those underlying the various error contributors, remain to be consolidated;
- The precise procedure with which λ_{eff} is determined remains to be established (e.g., Eq. 3 in GAIA-CA-TN-UB-CJ-040-1, Eq. 4 in GAIA-CA-TN-UB-CJ-040-1, or ...);
- Optimal computation of the coefficients B_{λ}^{XP} , transforming the overall response in XP to the overall response in AF, remains to be investigated;
- Details of the absolute-flux calibration of BP/RP spectra remain to be established;
- The spectral-coverage overlap between BP and RP (in the region 640–680 nm) needs to be properly dealt with;

- The robustness of the λ_{eff} estimation remains to be established (prompt-particle events, spectrum-edge effects, decentering of objects in the BP/RP windows, across-scan dispersion variations, wavelength-dependent flux losses, for example due to windowing, etc.);
- The consequences of a potential bias in λ_{eff} remain to be investigated;
- The consequences of BP/RP spectrum overlap resulting from crowding remain to be investigated;
- ...

6 Conclusion

The preliminary, exploratory study described here — which is still being developed at Lund Observatory by Lennart Lindegren and Oskar Svensson — suggests that typical chromaticity residual calibration errors $\sigma_{\text{cal,chrom}}$ of $5 \mu\text{as}$ for bright stars ($G \lesssim 13 \text{ mag}$) and $80 \mu\text{as}$ for faint stars ($G = 20 \text{ mag}$) are in reach (Table 1). These calibration residuals, applicable at the single-CCD level, are typically 3% of the photon-statistical centroiding error σ_{ξ} for a single CCD for stars fainter than 15-th magnitude and 6–10% at the bright end ($G \lesssim 13 \text{ mag}$). These values therefore seem to confirm that MRD requirement SCI-340, requiring “end-of-mission [chromaticity calibration] residuals of less than $2.5 \mu\text{as}$ for objects with $V = 15 \text{ mag}$ and $30 \mu\text{as}$ for objects with $V = 20 \text{ mag}$ ”, can be met. The method described here is, nonetheless, not necessarily the approach that will be used in the Data Processing and Analysis Consortium (DPAC) and numerous issues (some major, others minor) remain to be investigated.